Outline of my talk

1) Axion BEC: a model beyond CDM.

2) Production and Detection of Axion-like Particles by Interferometry.

Axion BEC: a model beyond CDM

Based on:

Bose-Einstein Condensation of Dark Matter Axions, P. Sikivie, Q. Yang, Phys. Rev. Lett. 103 (2009) 111301.

Cosmic Axion Thermalization, O. Erken, P. Sikivie, H. Tam, Q. Yang, Phys. Rev. D 85 (2012) 063520.

Axion Dark Matter and Cosmological Parameters, O. Erken, P. Sikivie, H. Tam, Phys. Rev. Lett. 108 (2012) 061304.

From the θ vacuum to axions

- A field configuration of QCD vacuum:

$$A_\mu = i/g U \partial_\mu U^\dagger$$

with a winding number n

$$n = -\frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} Tr[(U\partial_i U^\dagger)(U\partial_j U^\dagger)(U\partial_k U^\dagger)]$$

cannot be smoothly deformed into others with a different winding number without passing energy barriers.

- However these field configurations with different winding numbers can tunnel to each other due to instantons.

$$< n2|H|n1 >\sim e^{-S} \neq 0$$

- So the physical vacuum of QCD has to include field configurations with all possible winding numbers thus has the form:

$$|\omega >= \sum_n e^{in\theta}|n >.$$

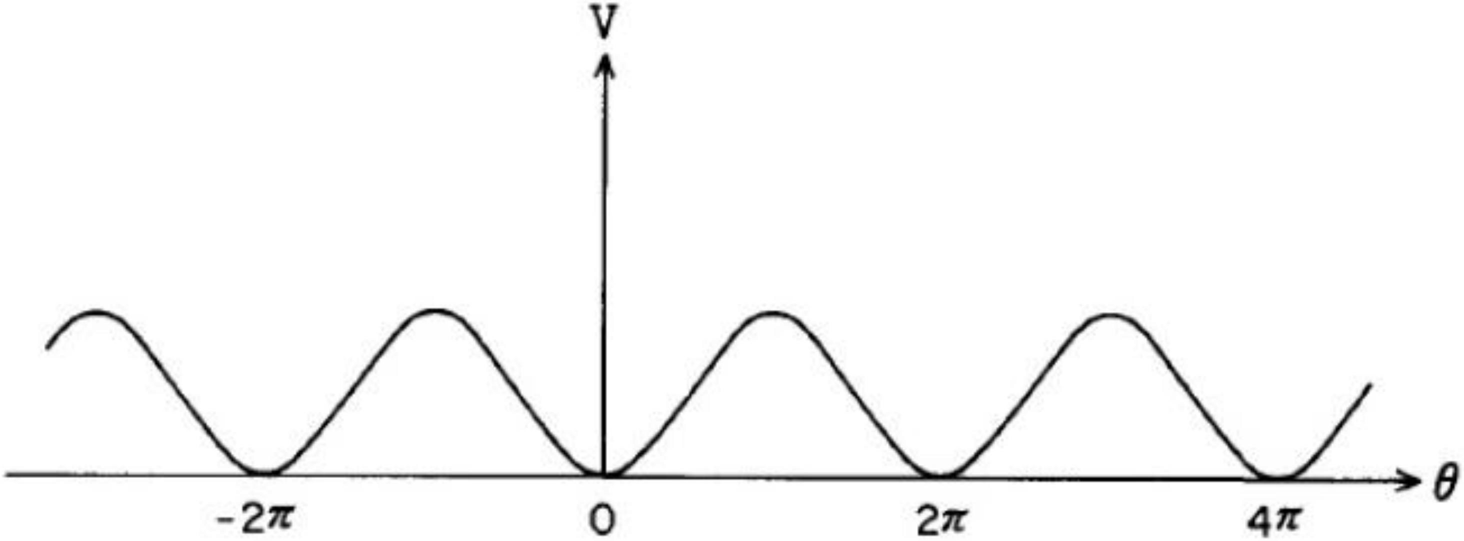
- The θ vacuum gives an additional potential term:

$$
\begin{aligned}
<\theta_1|e^{-Ht}|\theta> &= \sum_{n_1}\sum_{n} e^{-i(n_1\theta_1-n\theta)} <n_1|e^{-Ht}|n> \\
&= \sum_{n_1} e^{-in_1(\theta_1-\theta)} \sum_{n_1-n} \int [DA_\mu]_{n_1-n} exp[-\int d^4xL - i(n_1-n)\theta] \\
&= \delta(\theta_1-\theta) \int [DA_\mu]_q exp[-\int d^4x(L + \frac{\theta}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu a})],
\end{aligned}
$$

Where $q = n_1 - n$ is the Pontryagin index.

- This term violates CP invariance if $\theta \neq 0$
- The measurement of electric dipole moment of neutron gives an upper limit: $|\theta| \leq 10^{-9}$

The energy is minimized when $\theta = 0$ but since θ is a parameter, it does not automatically go to zero.



The energy due to the vacuum angle

One can introduce the $U(1)_{PQ}$ symmetry which is spontaneously broken

$$L = -1/4g^2\, Tr(G_{\mu\nu}G_{\mu\nu}) + \sum \bar{q}_(D_\mu \gamma_\mu + m_i)q_i$$
$$+ \theta/32\pi^2\, TrG_{\mu\nu}\tilde{G}_{\mu\nu} + 1/2\partial_\mu a \partial^\mu a + a/(f_a 32\pi^2)\, TrG_{\mu\nu}\tilde{G}_{\mu\nu},$$

$\theta + a/f_a$ relaxes to zero due to it is the lowest energy configuration and the strong CP problem is solved.

An example: the KSVZ axion

- One introduces an new complex scalar field σ and a new heavy quark Q that carry the PQ charge.

$$U(1)_{PQ}: \quad a \to a + f_a\alpha$$
$$\sigma \to exp(iq\alpha)\sigma$$
$$Q_L \to exp(iq\alpha/2)Q_L$$
$$Q_R \to exp(-iq\alpha/2)Q_R$$

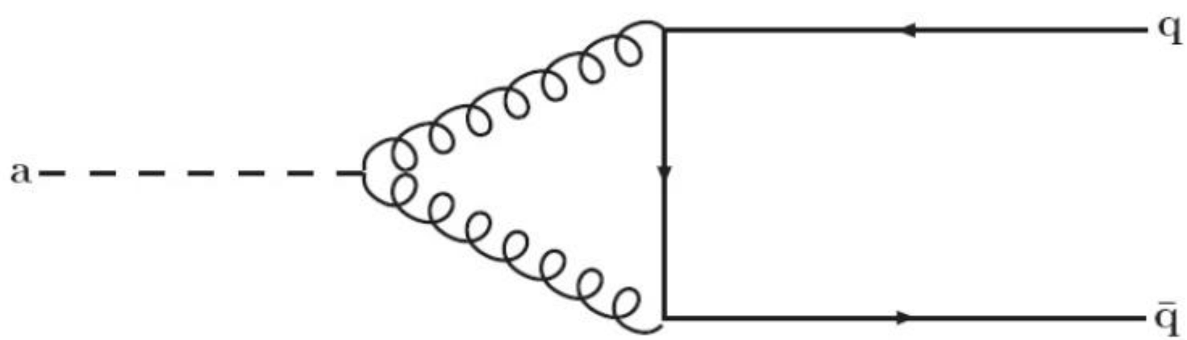
$$L_{Yu} = -fQ_L^\dagger \sigma Q_R - f^* Q_R^\dagger \sigma^* Q_R$$

$$V = -\mu_\sigma^2 \sigma^* \sigma + \lambda_\sigma (\sigma^* \sigma)^2$$

=> $\sigma = (v + \rho) exp(i\frac{a}{v})$.

$a/f_a \; TrG_{\mu\nu}\tilde{G}_{\mu\nu}$ term arises due to the ABJ anomaly.

- After QCD phase transition, the “PQ” Nambu-Goldstone boson acquires mass due to instanton effects, hence becoming a pseudo-Nambu-Goldstone boson, the “axion”.



The cold axions are produced by the misalignment mechanism (J. Preskill, M. Wise and F. Wilczek, Phys. Lett. B120 (1983)127; L. Abbott and P. Sikivie, Phys. Lett. B120 (1983) 133, M. Dine and W. Fischler, Phys. Lett. B120 (1983) 137.)

$$n_a \approx \frac{f_a^2}{2t_1}\left(\frac{a_1}{a_0}\right)^3$$

where t_1 is defined by $m(t_1)t_1 = 1$.
Therefore the cold axions have properties:

Small velocity dispersion: $\Delta v \sim [a(t_1)/a(t)] \cdot 1/mt_1$

High physical space density: $n(t) \sim \frac{4 \cdot 10^{47}}{\mathrm{cm}^3}\left(\frac{f}{10^{12}\mathrm{GeV}}\right)^{5/3}\left(\frac{a(t_1)}{a(t)}\right)^3$

High phase space density:

$$\mathcal{N} \sim n \frac{(2\pi)^3}{\frac{4\pi}{3}(m\delta v)^3} \sim 10^{61} \left(\frac{f}{10^{12}\ \mathrm{GeV}}\right)^{\frac{8}{3}}$$

Particle number is effectively conserved:

$$g_{a\gamma\gamma} \sim 10^{-22}\ \mathrm{eV}^{-1}$$

Bose-Einstein Condensation

- Results from the impossibility to allocate additional charges to the excited states for given temperature.
- Axion particle number is effectively conserved and is the relevant charge.
- Cold axions’ effective temperature is below the critical temperature which is very high due to their high number density.

If cold axions thermalize?

- We derive evolution equations for the out of equilibrium system, as an expansion in powers of the coupling strength.
- The axion field can be quantized as:

$$\varphi(\vec{x}, t) = \sum_{\vec{n}} \left[a(t)_{\vec{n}} \; \Phi_{\vec{n}}(x) \; + \; a^{\dagger}_{\vec{n}}(t)\Phi^{\star}_{\vec{n}}\right]$$

One solves the Heisenberg equation perturbatively,

$$\dot{a}_{\vec{n}} = i[H, a_{\vec{n}}]$$

where:

$$a_{\vec{n}}(t) = e^{-i\omega_{\vec{n}} t}(A_{\vec{n}} + B_{\vec{n}}(t)) + \mathcal{O}(\Lambda^2)$$

$$B_{\vec{n}}(0) = 0,\ A_{\vec{n}} = a_{\vec{n}}(0)$$

One gets:

$$\begin{aligned}
\dot{N}_{\vec{n}}(t) &= [-i \sum_{\vec{i},\vec{j},\vec{k}} \Lambda^{\vec{j},\vec{k}}_{\vec{n},\vec{i}}\, A^\dagger_{\vec{n}}\, A^\dagger_{\vec{i}}\, A_{\vec{j}}\, A_{\vec{k}}\, e^{-i\Omega t} + h.c.] \\
&+ [\sum_{\vec{i},\vec{j},\vec{k}} |\Lambda^{\vec{j},\vec{k}}_{\vec{n},\vec{i}}|^2 \{N_{\vec{i}} N_{\vec{j}} (N_{\vec{k}} + 1)(N_{\vec{n}} + 1) - N_{\vec{k}} N_{\vec{n}} (N_{\vec{i}} + 1)(N_{\vec{j}} + 1)\} \frac{sin(\Omega t)}{\Omega} \\
&+ off - diagonal\ \ 2_{\mathrm{nd}}\ \ order\ \ terms] + \mathcal{O}(\lambda^3)\ \ .
\end{aligned}$$

where $\Omega \equiv \omega_1 + \omega_2 - \omega_3 - \omega_4$

- For most cases, the first order terms along with the off diagonal second terms average to zero, the second order terms yield the Boltzmann equation.

- For cold axions due to high occupation number and extremely small energy dispersion, the first order terms no longer average to zero and dominate. Considering the high occupation numbers of axion states, we can replace the operators $A^\dagger$ and A with complex numbers whose magnitude is of order $\sqrt{N_{\vec{n}}}$.

The first order terms lead to the estimate of thermalization rate :

$$\Gamma_g \sim 4\pi G n m^2 l^2,$$

so cold axions thermalize after the photon temperature drops to 500eV. Future calculations show that cold axions remain thermalize when they falls into galactic halos. Cold axions may also thermalize with photons at equality. (uncertainties are due to $\Gamma_\gamma / H \sim 1$ barely satisfied at equality.)

Axion BEC in non-linear regime of structure formation (i.e. cold axions fall into galactic halo)

- In the galactic halo BEC has the property

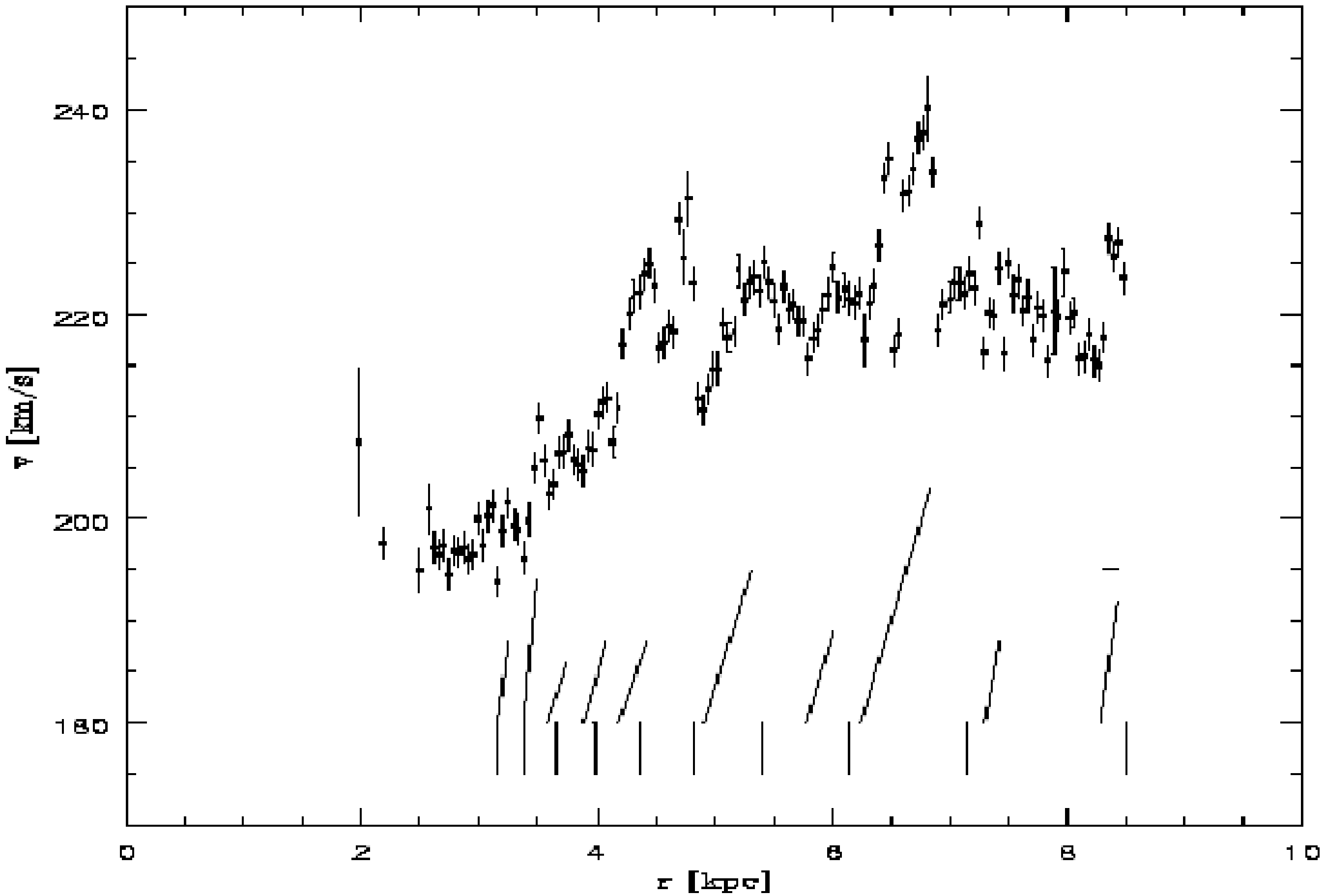
$\vec{\nabla} \times \vec{v} \neq 0$ by appearance of vortices.

- Collisionless particles have the property:

$$\vec{\nabla} \times \vec{v} = 0$$

- Axion BEC is consistent with the inner tricusp ring caustics which have observational indications.

Inner Galactic rotation curve



from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)

Implications for cosmological parameters

- Axions and photons may reach thermal contact at equality (between BBN and recombination). Energy conservation implies :

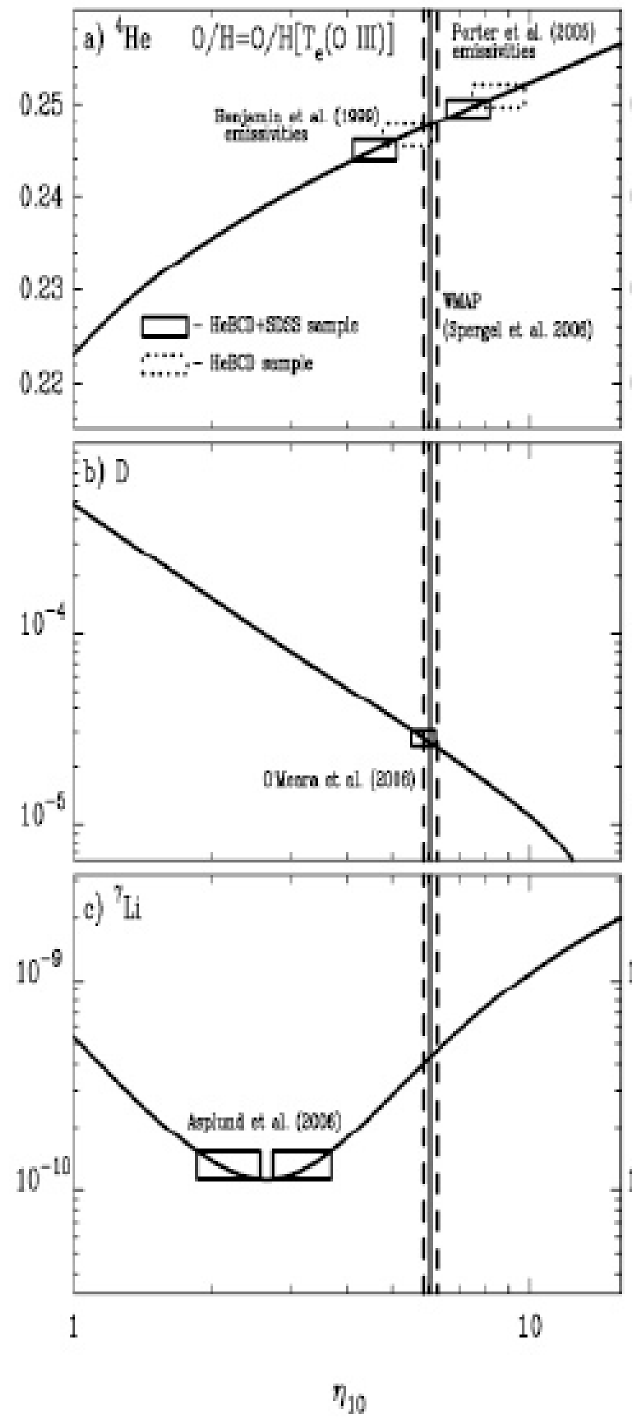
$$T_{\gamma f} = (\tfrac{2}{3})^{1/4} T_{\gamma i} = 0.9036 T_{\gamma i}$$

$$\eta_{10,\mathrm{BBN}} = \left(\frac{2}{3}\right)^{3/4} \eta_{10,\mathrm{WMAP}} = 4.57 \pm 0.11$$

- The effective neutrino number will be:

$$N_{\mathrm{eff}} = 6.77$$

The conflict between observations and predictions. Solid curves show predictions of the standard big bang nucleosynthesis, solid and dashed vertical lines indicate the value of η and its 1 σ deviations from WMAP. Boxes show the observed light element abundances and their 1 σ deviations. From Y. I. Izotov, T. X. Thuan and G. Stansinska[9]



The major drawback with D is that its abundance is inferred from a very small set of (seven) spectra of QSO absorption line systems. Also, since D is more easily destructible than 7Li, it is conceivable that unknown stellar processes further deplete D.

Li is inferred from a large number of measurements, which are more-or-less consistent.

Conclusions

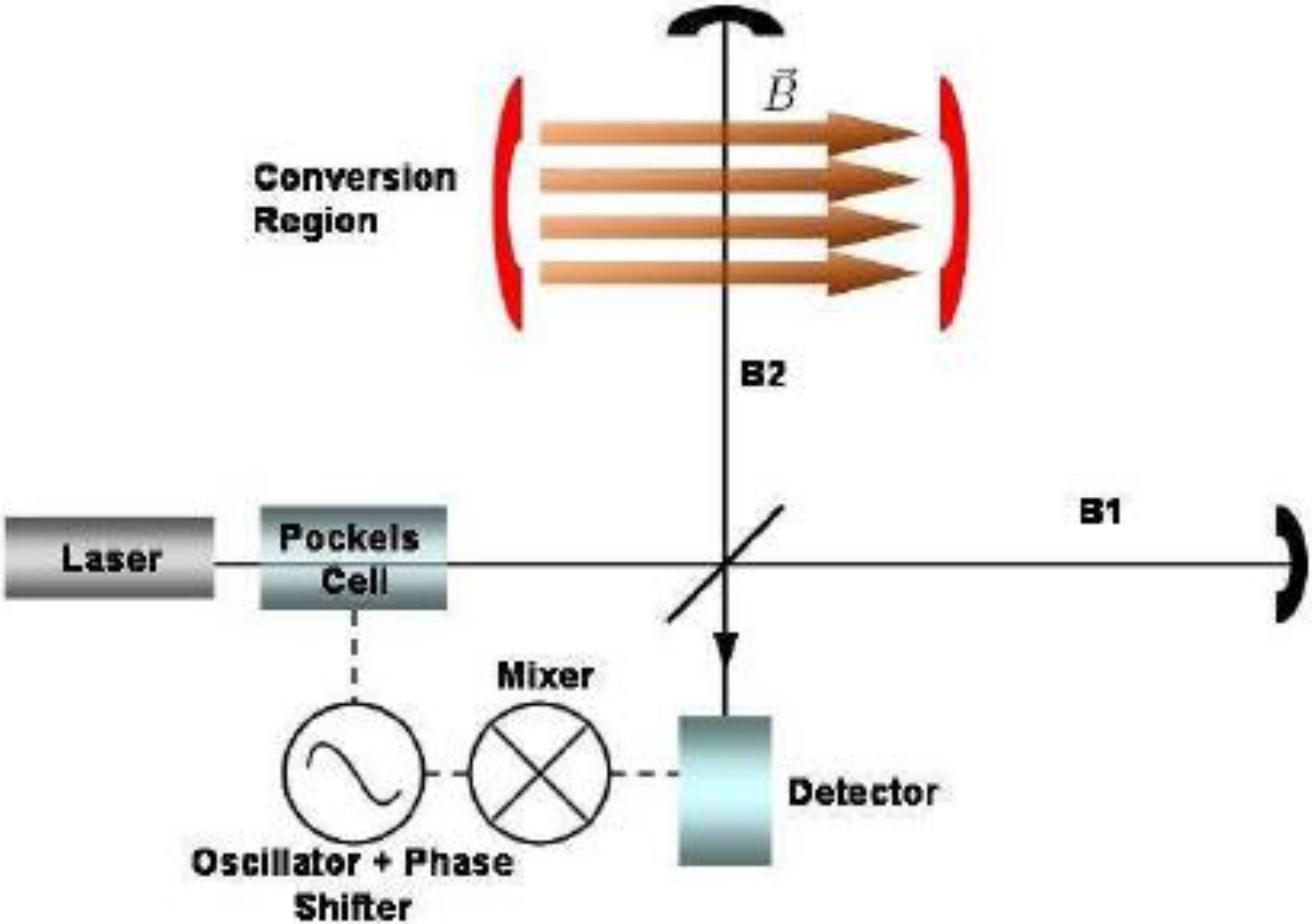
- Cold axions differ from ordinary CDM candidates in structure formation due to high phase space density and small energy dispersion.
- Cold axions form a BEC after photon temperature drops to 500eV.
- In galactic halo cold axions may give us a way to distinguish them from other CDM candidates.
- Cold axions may thermalize with other species therefore change the cosmological parameters.

Production and Detection of Axion-like Particles by Interferometry.

- Based on:

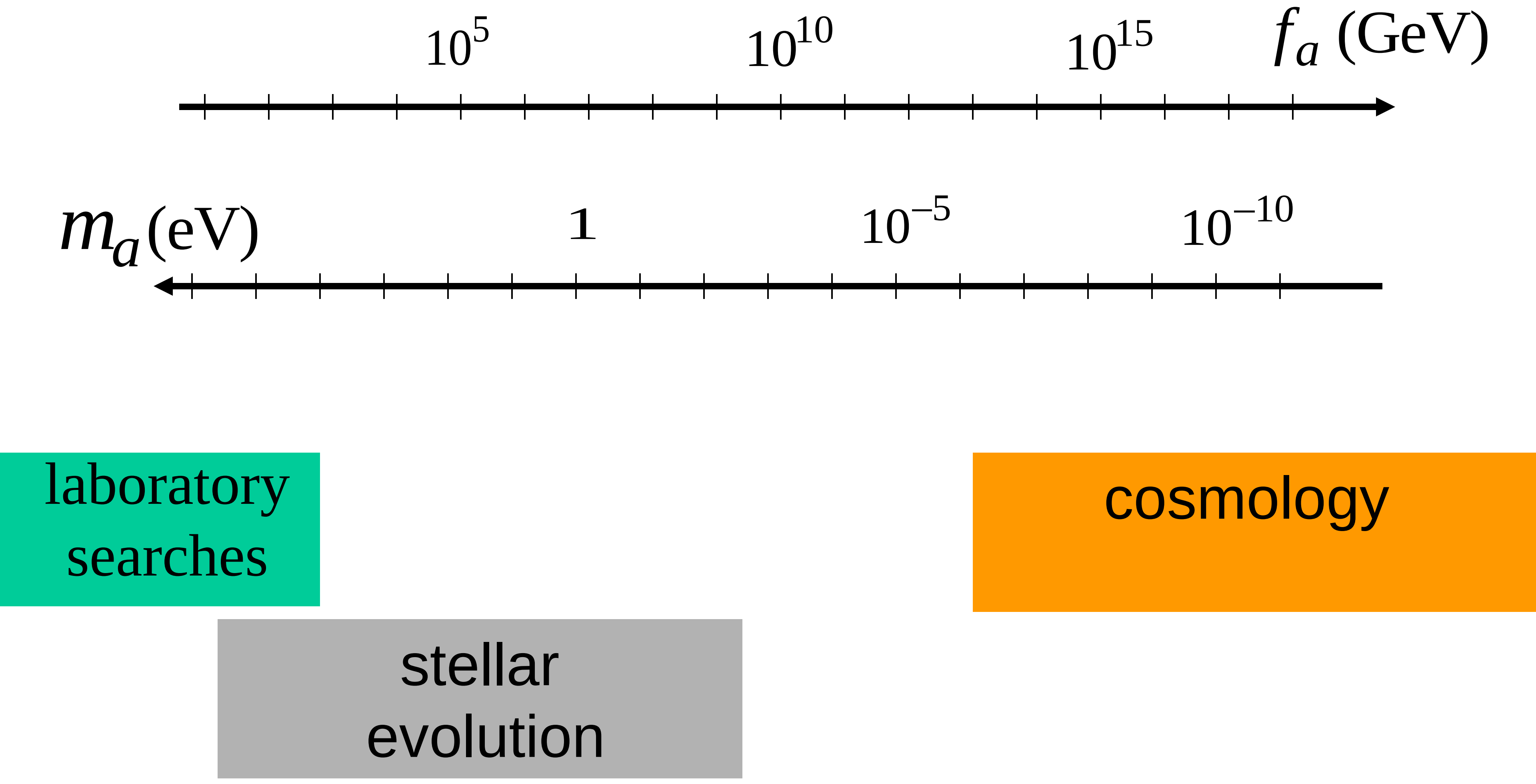
H. Tam, Q. Yang, arxiv.org/abs/1107.1712

To appear in Phys. Lett. B.



$\vec{B}$
Conversion Region
B2
B1
Laser
Pockels Cell
Mixer
Detector
Oscillator + Phase Shifter

The remaining axion mass and decay constant window



Magnitude of signals

for $\omega \sim \mathrm{eV}$ $B \sim 10\mathrm{T},\ L \sim 10\mathrm{m}$ $g_{a\gamma\gamma} \sim 10^{-11}\mathrm{GeV}^{-1}$

- Axions or ALPs:

Laser beam amplitude decreases:

$$\delta A_{\gamma\to a} = \frac{A\eta_{\gamma\to a}}{2} \approx \frac{g_{a\gamma\gamma}^2 B^2 L^2 A}{8} \qquad \delta A/A \approx 10^{-18}$$

Laser beam phase shifts:

$$\delta\theta \approx \frac{g_{a\gamma\gamma}^2 B^2 \omega_\gamma^2}{m_a^4}\left(\frac{m_a^2 L}{2\omega_\gamma} - \sin\left(\frac{m_a^2 L}{2\omega_\gamma}\right)\right)$$

if $m_a \ll m_0 \equiv \sqrt{2\pi\omega_\gamma/L}$, phase shift is negligible.

Magnitude of signals

- QED effective interaction terms:

$$\frac{\alpha^2}{90m_e^4}[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2])$$

- Phase shift due to QED effective interaction: (assuming same experimental parameters as in the last slide)

$$\mathcal{O}(10^{-14})$$

Amplitude modulation

- Carrier with sidebands is

$$\vec{E}_{in} = \vec{E}_0(1 + \beta \sin \omega_m t)e^{i\omega t}$$

At the dark fringe of carrier the signal is

$$\vec{E}_{carrier} = \frac{e^{i(\omega t + 2kL)}}{2}(\frac{\delta A}{A} + i\delta\theta)\vec{E}_0$$

Amplitude modulation

- The signal interferences with the side band, and the out put power at the detector is

$$
\begin{aligned}
P \;=\; & P_{in}\left\{ \frac{(\delta A/A)^2 + \delta\theta^2}{4} + \frac{\beta^2(4 - 4\frac{\delta A}{A} + \frac{\delta A^2}{A^2} + \delta\theta^2)}{2} \right. \\
+ \; & \beta(2\frac{\delta A}{A} - \frac{\delta A^2}{A^2} + \frac{\delta\theta^2}{2})\cos\left[\omega_m\left(t + \frac{2L}{c}\right)\right] \\
+ \; & \left. \frac{\beta^2(4 - 4\frac{\delta A}{A} + \frac{\delta A^2}{A^2} + \delta\theta^2)}{2}\cos\left[2\omega_m\left(t + \frac{2L}{c}\right)\right] \right\}
\end{aligned}
$$

Phase modulation

- If phase modulation is employed, the signal due to phase shift of laser beam is of first order and the signal due to amplitude decreasing of laser beam is of second order.

- Good for measuring the QED effect.

Intrinsic noise

- Due to shot noise

  Signal/noise= $(g_{a\gamma\gamma}BL)^2N/\sqrt{N}$

=> $g_{a\gamma\gamma,max} \sim (BL)^{-1}N^{-1/4}$

However optical delay line or F-P cavity enhances the signal by a factor of n, then the sensitivity of $g_{a\gamma\gamma}$ is boosted by a factor of n^(1/2) vs. n^(1/4) in the photon regeneration experiment.

Comparing with the resonantly enhanced photon regeneration

- Do not need to synchronize two cavities.
- Doubled conversion length, so signal is boosted by a factor of 4.

Comparing with the polarimetry (sensitivity of polarimetry is severely limited by the intrinsic birefringence of the optical devices.)

- Intrinsic noise is shot noise which is well understood.
- Future boost of sensitivity is possible by employing squeezed light.

A practical concern

- One can rotate the polarization of laser beam to see if the signal magnitude changes as expected. By using this way one excludes possible signal spoiling due to magnetic force optical device interactions.

Conclusions

using a 10 W laser $(\lambda = 1\mu\text{m})$ $B \sim 10\text{T},\ L \sim 10\text{m}$

- QED effect can be observed even without employing power recycling technical (phase modulation is required).

- If n=10^3, after 240 hours running, one can reach: $g_{a\gamma\gamma} > 2.8 \times 10^{-10}\text{GeV}^{-1}$
  with 5σ significance. If one uses the squeezed light method, future improvement can be achieved (amplitude modulation is required).